Tuning Locked Inflation: Supergravity versus Phenomenology

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We analyze the cosmological consequences of *locked inflation*, a model recently proposed by Dvali and Kachru that can produce significant amounts of inflation without requiring slow-roll. We pay particular attention to the end of inflation in this model, showing that a secondary phase of *saddle inflation* can follow the locked inflationary era. However, this subsequent period of inflation results in a strongly scale dependent spectrum that can lead to massive black hole formation in the primordial universe. Avoiding this disastrous outcome puts strong constraints on the parameter space open to models of locked inflation.

I. INTRODUCTION

The inflationary paradigm [1] provides a compelling account of early universe cosmology. The universe emerges from the inflationary phase with large-scale homogeneity and endowed with a nearly scale invariant spectrum of density fluctuations, consistent with current observations. Despite these impressive phenomenological achievements, designing successful models of inflation within supergravity and string theory has proven to be a frustratingly difficult task [2]. In spontaneously broken supergravities lifted flat directions are natural inflaton candidates. However, the various moduli have (stable, protected) masses m of order H, the Hubble constant during inflation [3, 4]. This perversely spoils slow-roll inflation since the slow-roll condition, $\eta \sim m^2/H^2 \ll 1$, is then violated. In other words, the generic outcome in supergravity theories is $\eta \gtrsim \mathcal{O}(1)$. This so-called η-problem is encountered, for instance, in attempts to embed inflation in the stringy landscape [5]. Recent developments, however, indicate that a simple change to the Kähler potential might alleviate this problem [6].

In a recent paper, Dvali and Kachru [7] (henceforth, DK) introduced *locked inflation* as a possible way out of this dilemma. Its distinguishing feature is that it does away with the slow-roll constraints. Instead locked inflation relies on the rapid oscillations of one scalar field which lock a second field at the top of a saddle point. The potential energy at the saddle then drives inflation. At the very least, this model is an intriguing alternative to slow-roll inflation and, if consistent, overcomes the hurdles faced by slow-roll inflation in supergravity and string theories. Locked inflation needs no intrinsically small parameters, although it does exploit – like most two-field models – the ratio of the two widely separated scales. The existence of widely separated scales is not a new tuning, however, as this hierarchy must be explained even in the absence of inflation.

In this paper, we examine the termination of locked inflation and the subsequent evolution of the universe. It is perhaps natural to assume that inflation ends as soon as the field-point is no longer trapped at the saddle point. However, for the parameter values natural in broken supergravities, we find that this is not necessarily the case. Instead, the universe can undergo a second period of inflation as the field point moves orthogonally to the direction about which it was previously oscillating. We dub this phase *saddle inflation*.

Saddle inflation has potentially disastrous observational consequences for the DK scenario. As we will show, modes that leave the horizon at the onset of saddle inflation typically have an amplitude of order unity and thus give rise to a phenomenologically dangerous number of massive black holes when they re-enter the horizon during the subsequent radiation or matter-dominated eras. The formation of these black holes must thus be avoided at all costs. There are two ways to do so. One is to demand that there be no saddle inflation at all. This requires that the (tachyonic) mass of the saddle field be much larger than H, or $\eta \gg 1$. The other way out of the black hole problem is to make the secondary phase of saddle inflation last long enough to move the dangerous range of scales outside the present cosmological horizon. This renders the prior period of locked inflation unobservable, although the latter remains useful as a mechanism for resolving the initial conditions problem related to the onset of inflation. Long saddle inflation naturally occurs for $\eta \ll 1$.

The generation of large perturbations at a saddle point of the potential has been discussed for general two-field models by Garcia-Bellido, Linde and Wands [8], and in the specific context of supernatural inflation by Randall, Soljacic and Guth [9]. Adopting the analyses of these earlier models to the specific case of locked inflation, we can readily deduce the phenomenological consequences associated with the end of locked inflation.

We conclude that a viable model of locked inflation requires either $\eta \gtrsim 30 - 1500$ (no saddle inflation), depending on the reheating temperature, or $\eta \lesssim 0.01 - 2.5$ (long saddle inflation), similarly depending on the reheating temperature. Thus, at a naive level, it appears that both models necessitate a similar degree of tuning.

It is noteworthy that long saddle inflation with $\eta \sim \mathcal{O}(1)$ is allowed for reheating temperature in the range $1-10^9$ GeV. Locked inflation only sets up the desired initial condition for saddle inflation in this case. Nevertheless, this is a successful inflationary model with supergravity-

inspired potential. As such, it is a clear candidate for embedding inflation in string theory and supergravity. To reproduce the observed scale invariant spectrum of density perturbations, however, it does require that the fluctuations arise from an alternative mechanism.

II. LOCKED INFLATION: A REVIEW

Consider the simple, two-field potential

$$\mathcal{V}(\Phi,\phi) = m_\Phi^2\Phi^2 + \lambda\Phi^2\phi^2 + M^4\left(1 - \frac{\eta}{4}\frac{\phi^2}{M_{Pl}^2}\right)^2 , \quad (1)$$

where $m_\Phi^2 \sim M^4/M_{Pl}^2$, λ is a dimensionless parameter of order unity, M is of order of the supersymmetry breaking scale, and $M_{Pl} = (8\pi G)^{-1/2}$ is the reduced Planck mass. The meaning of the dimensionless parameter η will soon become clear [10]. DK show that this potential produces locked inflation when the field-point oscillates rapidly in Φ direction, while $\phi \sim 0$. Eventually these oscillations are sufficiently damped to allow the field point to roll off the saddle point in the ϕ direction, thereby ending locked inflation.

As the universe inflates, the Hubble parameter H is nearly constant and given by $3H^2M_{Pl}^2 \approx M^4$. The number of e-folds of inflation generated while Φ oscillates equals

$$N_{locked} \approx \frac{4}{3}\ln\left(\frac{M_{Pl}}{M}\right) . \quad (2)$$

For $M = 1$ TeV, this gives $N_{locked} \approx 50$, and therefore one stage of locked inflation is sufficient in this case to account for the homogeneity and isotropy of our observable universe. Larger values of the small energy scale M, however, require multiple stages, each contributing N_{locked} e-folds of inflation. This "cascade" scenario is not unreasonable in string theory, as DK argue, since we expect that the field point will go through many saddle points in the stringy landscape before reaching the true vacuum of the theory.

Our numerical calculations confirm the estimate for N_{locked} given above. However, there is an additional constraint on m_Φ. If it is too small ($< 3H/2\sqrt{2}$), Φ will be overdamped, and no oscillations will occur. On the other hand, if m_Φ is too large ($> 10H/\sqrt{2}$), one can produce Φ particles via parametric resonance, and the kinetic energy will be rapidly drained from the Φ field, undermining locked inflation [11]. While this restricts the parameter range open to successful models of locked inflation, the natural expectation from supergravity that $m_\Phi \sim H$ is compatible with these constraints.

III. INFLATION AFTER LOCKED INFLATION

In the remainder of this paper, we focus on the field evolution immediately after locked inflation. In particular, we argue that it is possible for the universe to undergo further inflation for a wide range of parameter values. This effect is well known in two-field models [8] and is particularly important in supernatural inflation [9] where it produces a large "spike" in the density perturbation spectrum at small wavelengths.

Once a given stage of locked inflation ends, the field point rolls off in the ϕ-direction. By this time the oscillations in Φ can safely be ignored and, since $\Phi \approx 0$, the problem reduces effectively to a single-field model with potential

$$V(\phi) = M^4\left(1 - \frac{\eta}{4}\frac{\phi^2}{M_{Pl}^2}\right)^2 . \quad (3)$$

We derive conditions under which a subsequent inflationary phase will occur as ϕ rolls off, a phase which we henceforth refer to as *saddle inflation*. In general this phase will not satisfy the usual slow roll conditions of inflation, however. Indeed, near the top of the hill where $\phi \ll M_{Pl}$, the slow-roll parameter η_s is given by

$$|\eta_s| \equiv M_{Pl}^2\left|\frac{V_{,\phi\phi}}{V}\right| = \eta + \mathcal{O}\left(\frac{\phi}{M_{Pl}}\right) , \quad (4)$$

which is greater than unity for $\eta \gtrsim 1$. The virtue of writing the potential in Eq. (3) in terms of η is now clear, as the latter reduces to $|\eta_s|$ for small ϕ.

While the usual slow-roll expressions are not generally applicable here, we can nevertheless make use of a quadratic approximation. In natural inflation [12, 13] this is known as the *small angle* approximation. Its range of applicability is much greater than that of the slow-roll approximation, and it contains slow-roll as a limit. For small ϕ, V reduces to

$$V(\phi) \approx M^4 - \frac{1}{2}\frac{\eta M^4}{M_{Pl}^2}\phi^2 . \quad (5)$$

The dynamics of ϕ are thus governed by the equation of motion (recall that we approximate H to be constant)

$$\frac{d^2\phi}{dN^2} + 3\frac{d\phi}{dN} \approx 3\eta\phi , \quad (6)$$

where $N \equiv \ln a$ is the number of e-folds. The initial conditions for ϕ can be estimated as follows. At the end of the locked inflation, the effective mass in the ϕ direction is effectively zero, but the expected quantum fluctuation in ϕ is on the order of H. Thus, semiclassically we can assume $\phi_{init} \sim H$ [8] and that the initial velocity is zero. With these initial conditions, the solution to Eq. (6) is

$$\phi(N) = \frac{\phi_{init}}{2\delta}\left\{(\delta+1)\exp\left[\frac{3}{2}(\delta-1)N\right] + (\delta-1)\exp\left[-\frac{3}{2}(\delta+1)N\right]\right\} , \quad (7)$$

where $\delta \equiv \sqrt{1+4\eta/3}$. Let us assume that we will be able to show that this solution can correspond to an inflationary phase, which by definition lasts longer than

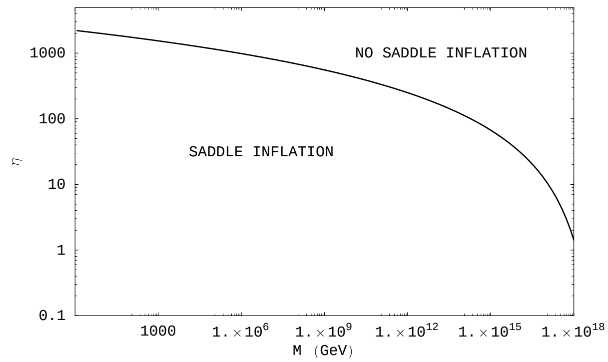

FIG. 1: The curve delimits those values of η and M (GeV) that lead to saddle inflation from those that do not.

one e-fold. Since $\delta > 1$, after one e-fold we can ignore the second term in Eq. (7) and write

$$\phi \approx \frac{\phi_{init}}{2\delta}(\delta+1)\exp(f(\eta)N)\,, \tag{8}$$

where

$$f(\eta) \equiv \frac{3}{2}\left\{\sqrt{1+\frac{4}{3}\eta}-1\right\}\,. \tag{9}$$

The function f will play an important role in the results to be derived below.

To show that this solution indeed yields an inflationary phase, consider

$$\epsilon = \frac{3}{2}(1+w)\,, \tag{10}$$

where w is the equation of state for ϕ, given by $w \equiv (\dot{\phi}^2-2V)/(\dot{\phi}^2+2V)$. In the slow-roll approximation, ϵ reduces to the usual slow-roll parameter $\epsilon_s \equiv M_{Pl}^2 V_{,\phi}^2/2V^2$. Inflation takes place if $w \approx -1$, or, equivalently, $\epsilon \ll 1$. Using Eq. (8), ϵ is seen to equal

$$\epsilon \approx \frac{1}{2}f^2(\eta)\frac{\phi^2}{M_{Pl}^2}\,. \tag{11}$$

Saddle inflation terminates when $\epsilon \sim \mathcal{O}(1)$ or when the quadratic approximation breaks down, whichever happens first. For $\eta \gtrsim 1$, the relevant range for the discussion below, inflation ends when $\epsilon \sim \mathcal{O}(1)$. From Eq. (11), this occurs when the field reaches $\phi_{end} \approx \sqrt{2}f^{-1}(\eta)M_{Pl}$. Substituting in Eq. (8), we find that the total number of e-folds, N_{saddle}, generated during saddle inflation is given by

$$\begin{aligned} N_{saddle} &= \frac{1}{f(\eta)}\ln\left(\frac{\sqrt{2}(2f(\eta)+3)}{f(\eta)(f(\eta)+3)}\frac{M_{Pl}}{\phi_{init}}\right) \\ &\approx \frac{1}{f(\eta)}\ln\left(\frac{2\sqrt{6}}{f(\eta)}\frac{M_{Pl}^2}{M^2}\right)\,. \end{aligned} \tag{12}$$

The discussion thus far only applies if $N_{saddle} \gtrsim 1$; otherwise there is no inflation. Conversely, there will be no saddle inflation if $N_{saddle} \lesssim 1$. Using Eqs. (12) and (9) we can derive a necessary and sufficient condition for the existence of a period of saddle inflation by working out the minimum value of η for which $N_{saddle} < 1$. This bound is plotted in Fig. 1. As illustrative examples, the bound on η for two physically interesting values of M is:

$$\begin{aligned} \eta_{min} &\approx 1500 \qquad \text{for} \quad M \sim 1\ \text{TeV} \\ \eta_{min} &\approx 30 \qquad \text{for} \quad M \sim 10^{16}\ \text{GeV}\,. \end{aligned} \tag{13}$$

To summarize, we have shown that some inflation will follow the period of locked inflation if $\eta \lesssim 10-10^3$, depending on the choice of reheating temperature M.

IV. PERTURBATION SPECTRUM AND BLACK HOLE FORMATION

We next compute the spectrum of density fluctuations generated during saddle inflation. In Newtonian gauge, the linearized equation for the k-mode u_k of the gauge-invariant perturbation variable u, related to the Newtonian potential by $\Phi_{Newton} = uH(d\phi/dN)$, is [14]

$$\frac{d^2u_k}{dN^2}+\frac{du_k}{dN}+\left(\frac{k^2}{a^2H^2}-\beta(N)\right)u_k = 0\,; \tag{14}$$

$$\beta(N) = \frac{1}{2}\left\{\frac{d\ln\epsilon}{dN}+\frac{1}{2}\left(\frac{d\ln\epsilon}{dN}\right)^2-\frac{d^2\ln\epsilon}{dN^2}\right\}+\mathcal{O}(\epsilon)\,,$$

where we recall that $a(N) \equiv \exp(N)$. The only approximation made in deriving Eq. (14) is that the universe is inflating, that is, $\epsilon \ll 1$. No assumption was made about the time-dependence of ϵ, however.

Substituting Eq. (11) and using Eq. (8), we find

$$\beta \approx f(\eta)(1+f(\eta))\,. \tag{15}$$

In particular, since β is nearly constant, Eq. (14) becomes analytically solvable. Choosing the usual Bunch-Davies vacuum in the short-wavelength limit, we get [14]

$$\begin{aligned} k^{3/2}\,u_k &= \frac{\sqrt{\pi}}{4}\sqrt{\frac{k}{aH}}H_p^{(1)}\left(\frac{k}{aH}\right) \\ p(\eta) &\equiv \sqrt{f(\eta)(1+f(\eta))+1/4}\,, \end{aligned} \tag{16}$$

where $H_p^{(1)}$ is the Hankel function of the first kind of order p. In the long-wavelength limit, $k^2/a^2H^2 \ll \beta$, this gives

$$k^{3/2}|u_k| \approx \frac{\sqrt{\pi}}{2^{2-p}\sin(\pi p)\Gamma(1-p)}\left(\frac{k}{aH}\right)^{-p+1/2}\,, \tag{17}$$

from which we can read off the spectral index of the fluctuations:

$$n_s-1 \approx -2p(\eta)+1 = -\sqrt{1+4f(\eta)(1+f(\eta))}+1\,. \tag{18}$$

As a check, recall that in the slow-roll approximation we have $f(\eta) \approx \eta \ll 1$, and thus $n_s - 1 \approx -2\eta$. We see from Eq. (4) that this agrees with the usual slow-roll expression for the spectral index n_s.

The amplitude of the density perturbations is naturally expressed in terms of ζ_k, the curvature perturbation on comoving hypersurfaces [15]. This gauge-invariant variable is related to u_k by

$$\zeta_k = \frac{H}{\epsilon a}\frac{d}{dN}\left(a\frac{d\phi}{dN}u_k\right). \tag{19}$$

In single-field inflation ζ_k is nearly constant outside the horizon. Single-field inflation is a good approximation during saddle inflation since, as mentioned earlier, the second field, Φ, is essentially inert. Thus it suffices to evaluate ζ_k at horizon-crossing, that is, at $k = aH$. Substituting Eq. (17), and using Eqs. (8) and (11), we obtain

$$k^{3/2}\zeta_k = \frac{\sqrt{\pi}[f(\eta)+p(\eta)+1/2]}{2^{2-p}\sin(\pi p)\Gamma(1-p)}\left.\frac{\sqrt{2}HM_{Pl}}{\sqrt{\epsilon}}\right|_{k=aH}, \tag{20}$$

where the subscript "$k = aH$" denotes horizon-crossing.

Let us evaluate this quantity for the first mode to leave the horizon during saddle inflation. This mode freezes out when $\phi = \phi_{init} \sim H$, and, therefore, from Eq. (11), $\sqrt{\epsilon}_{k=aH} \approx f(\eta)H/\sqrt{2}M_{Pl}$. Substituting in Eq. (20), we find that it has an amplitude

$$\begin{aligned}\frac{(k^{3/2}\zeta_k)_{init}}{M_{Pl}^2} &= \frac{\sqrt{\pi}[f(\eta)+p(\eta)+1/2]}{2^{2-p(\eta)}\sin(\pi p(\eta))\Gamma(1-p(\eta))}\frac{2H}{\phi_{init}f(\eta)}\\ &\approx \frac{\sqrt{\pi}[f(\eta)+p(\eta)+1/2]}{2^{2-p(\eta)}\sin(\pi p(\eta))\Gamma(1-p(\eta))}\frac{2}{f(\eta)}.\end{aligned} \tag{21}$$

Since p and f are both functions of η, this entire expression depends solely on η. Moreover, it is easily seen that it is numerically always greater than unity.

Therefore, over the range of scales corresponding to the first few modes to leave the horizon during saddle inflation, the amplitude of density fluctuations is of order unity. When these modes re-enter the horizon after reheating, there is a probability of roughly a half that the overdensity will collapse and form a black hole [16]. The issue of black hole formation is not unique to locked inflation, and has been been studied in the context of supernatural inflation in particular [8]. In the case of supernatural inflation, the problem of black hole formation can be avoided in two ways: either η must be tuned to sufficiently large values to prevent saddle inflation from occurring; or the scale of inflation must be sufficiently high so that the black holes evaporate well before nucleosynthesis. In the context of locked inflation, as we will now argue, this latter option fails.

V. CONSEQUENCES FOR LOCKED INFLATION

Consider first the model where a single stage of locked inflation is sufficient to solve the horizon and flatness problems, i.e. we choose $M \sim 1$ TeV. Suppose that this stage of locked inflation is followed by a period of saddle inflation, which, from Eq. (13), requires $\eta \lesssim 1600$. As mentioned above, when the mode corresponding to ϕ_{init} re-enters the horizon, a black hole is likely to form. It suffices to focus on the case where only one e-fold or so is generated during saddle inflation since a longer inflationary phase will only make the black hole larger. Then, the Schwarzschild radius of the black hole formed, $R_S \sim \delta\rho/H^3M_{Pl}^2$, is of the order of the Hubble radius at reheating, $H^{-1} \sim M_{Pl}/M^2$. That is, the black hole has mass

$$M_{BH} \sim \frac{M_{Pl}^3}{M^2}. \tag{22}$$

For $M \sim 1$ TeV, this gives $M_{BH} \sim 10^{26}$ g, or roughly the mass of the Earth! These massive black holes have a lifetime longer than the present age of the universe and thus dominate the evolution of the universe well before nucleosynthesis.

Equation (22) suggests that this problem could be avoided by making the small scale M sufficiently large. Indeed, it seems this would make M_{BH} sufficiently small such that the black holes would evaporate well before nucleosynthesis. This is indeed the avenue taken in supernatural inflation where one imposes $M \gtrsim 10^{11}$ GeV [8].

However, locked inflation is different. This is because larger values of M require multiple stages of locked inflation, as in the cascade model proposed by DK. Suppose each stage generates ΔN_{locked} e-folds of inflation, with ΔN_{locked} given by Eq. (2). Moreover, suppose that a substantial fraction of all stages are followed by a short period of saddle inflation. As argued in Sec. IV, to each phase of saddle inflation there corresponds a range of scales for which black holes will form when the corresponding modes re-enter the horizon. The typical wavelength of the largest black hole thus formed will be of order

$$M_{BH} \sim \frac{M_{Pl}^2}{H_0}\left(\frac{M}{M_{Pl}}\right)^{8/3}, \tag{23}$$

where H_0 is today's Hubble constant. For $M = 10^{11}$ GeV, say, this gives $M_{BH} \approx 10^{35}$ g. Therefore, in locked inflation with short stages of saddle inflation, the generic mass of black holes produced *increases* with M, making the problem worse.

VI. HOW TO AVOID BLACK HOLE FORMATION

As mentioned earlier, one way out of the black hole problem is simply to avoid any saddle inflation. This is

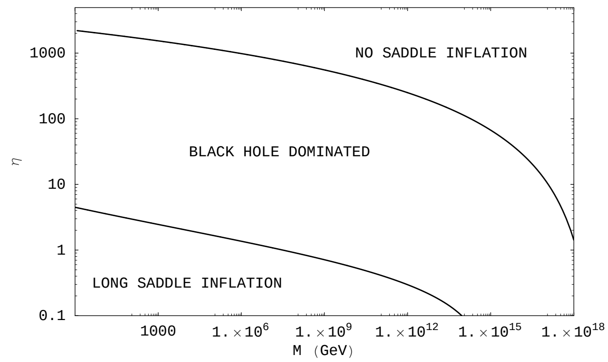

FIG. 2: Phenomenologically allowed choices for η and M (GeV). The region "Long Saddle Inflation" corresponds to the values of η and M for which saddle inflation lasts long enough to push the dangerous range of modes outside the present horizon. Unacceptable black hole production at horizon reentry excludes the middle region.

the case if (η, M) lie in the "No Saddle Inflation" region of Fig. 1.

An alternative approach is to impose that saddle inflation lasts long enough to push the dangerous range of modes well beyond the scale of our observable universe. In other words, modes with amplitude of order unity are still generated, but their wavelength is sufficiently long that they have not re-entered the horizon by the present time. In this case, however, all the modes within our observable universe are generated during saddle inflation, leaving locked inflation with no observational consequences.

In order to be phenomenologically viable, the long period of saddle inflation must satisfy two constraints. First, as mentioned above, it must last sufficiently long to push the dangerous modes outside our observable universe. Second, the spectrum of perturbations within our horizon must be consistent with observations. This latter constraint normally forces one to the slow-roll regime ($\eta \ll 1$). However, quantum fluctuations of the "inflaton" ϕ need not necessarily be responsible for the spectrum of density perturbations. In particular, let us suppose that that they are produced via the alternative mechanism of Dvali, Gruzinov and Zaldarriaga [17] (henceforth, DGZ). In this case, the spectrum is generated by the fluctuations of some other field χ. This field is assumed to be nearly massless during inflation, and, moreover, its vacuum expectation value determines the coupling constants between the inflaton and the various Standard Model fields.

Allowing for alternative sources of density perturbations, such as the DGZ mechanism, greatly expands the range of allowed models. For instance, the spectral index for the fluctuations generated a la DGZ is [17]

$$(n_s - 1)_\chi = -2\epsilon \,. \tag{24}$$

In contrast with the familiar slow-roll result of $(n_s - 1)_\phi = -6\epsilon + 2\eta$, Eq. (24) is independent of η. Since ϵ is small during saddle inflation, it is therefore possible to satisfy the observational constraint $|n_s - 1| \lesssim 0.1$ from WMAP [18] and observations of large-scale structure without requiring $\eta \ll 1$. Similarly, the expression for the amplitude of the perturbations is different than in slow-roll inflation [17]. For our purposes, we shall assume that it is possible for χ to lead to density perturbations with amplitude of 10^{-5}, consistent with COBE data.

Nevertheless, fluctuations in the "inflaton" do contribute to the density perturbation spectrum, on top of the χ contribution. To be consistent with COBE data, these ϕ-generated fluctuations must be less than 10^{-5} in amplitude on scales comparable to the size of the universe today. That is, from Eqs. (11) and (20), we must impose

$$\frac{M^2}{f(\eta)\phi_0 M_{Pl}} \lesssim 10^{-5} \,, \tag{25}$$

where we have neglected a factor of order unity and used the relation $H \sim M^2/M_{Pl}$. Here ϕ_0 is the field value of when the mode corresponding to our universe today left the horizon during saddle inflation.

Let N_0 denote the difference in the number of e-foldings between the mode which corresponds to our present universe and the last mode to exit the horizon during saddle inflation. This is approximately given by the logarithm of the ratio of temperatures. Since the temperature is currently $T_0 = 2.723$ K, while at reheating it was of order M, we get

$$N_0 \approx \ln\left(\frac{M}{T_0}\right) \,. \tag{26}$$

For $\eta \lesssim 1$, the relevant range for this discussion, saddle inflation ends when the quadratic approximation breaks down. From Eq. (5), this occurs when the field reaches $\phi_{end} \approx M_{Pl}/\sqrt{\eta}$. Substituting this in Eq. (8), we obtain a second independent expression for N_0:

$$N_0 \approx \frac{1}{f(\eta)} \ln\left(\frac{M_{Pl}}{\sqrt{\eta}\phi_0}\right) \,. \tag{27}$$

The above expressions for N_0 together imply that

$$\frac{1}{\phi_0} \approx \frac{\sqrt{\eta}}{M_{Pl}} \left(\frac{M}{T_0}\right)^{f(\eta)} \,, \tag{28}$$

which, when substituted in Eq. (25), gives

$$\frac{\sqrt{\eta}}{f(\eta)} \left(\frac{M}{M_{Pl}}\right)^2 \left(\frac{M}{T_0}\right)^{f(\eta)} < 10^{-5} \,. \tag{29}$$

This condition on η and M ensures that the perturbations in ϕ are sufficiently small to be consistent with COBE data. The values of η and M which obey this condition lie in the "Long Saddle Inflation" region in Fig. 2.

A moment's thought reveals that this condition also ensures that saddle inflation lasts long enough to push the

dangerous modes beyond our present horizon. Indeed, recall from Eq. (18) that the spectral index for the ϕ-perturbations is given by $(n_s - 1)_\phi \approx -2\eta$, corresponding to a red spectrum. That is, larger wavelength modes have larger amplitude. Moreover, recall that the dangerous modes that eventually lead to black hole formation have amplitude of order unity. Since Eq. (25) constrains the amplitude on today's scales to be much less than unity, however, it follows that these modes must lie well beyond our horizon.

VII. DISCUSSION

We have shown that, in order for locked inflation to be phenomenologically viable, it must either: i) end without any subsequent saddle inflation; or ii) be followed by a long phase of saddle inflation. This constrains the (η, M) parameter space as illustrated in Fig. 2. In the former case, no black holes are formed. In the latter case, saddle inflation lasts sufficiently long to push the dangerous modes outside our observable universe. However, locked inflation would only serve the purpose of setting up the initial conditions for saddle inflation in this case, but would have no directly testable observational consequences in itself.

In case i), shown as "No Saddle Inflation" in the Figure, η and M are required to satisfy (see Eq. (12))

$$\begin{aligned} \frac{f^2(\eta)(f(\eta)+3)^2}{6(2f(\eta)+3)^2}\exp[2f(\eta)] &\gtrsim \frac{M_{Pl}^4}{M^4} \\ \overset{\eta\gg 1}{\Longrightarrow} \qquad \frac{\eta}{8}\exp[2\sqrt{3\eta}] &\gtrsim \frac{M_{Pl}^4}{M^4}, \end{aligned} \qquad (30)$$

which implies $\eta \gtrsim 30 - 1500$ for $M \approx 10^{16}$ GeV $-$ 1 TeV.

In case ii), shown as "Long Saddle Inflation" in the Figure, the bound on η and M is (see Eq. (25))

$$\begin{aligned} \frac{\sqrt{\eta}}{f(\eta)}\left(\frac{M}{M_{Pl}}\right)^2\left(\frac{M}{T_0}\right)^{f(\eta)} &< 10^{-5} \\ \overset{\eta\ll 1}{\Longrightarrow} \qquad \frac{1}{\sqrt{\eta}}\left(\frac{M}{T_0}\right)^{\eta} &< 10^{-5}\left(\frac{M_{Pl}}{M}\right)^2, \end{aligned} \qquad (31)$$

or $\eta \lesssim 0.01 - 2.5$ for $M \approx 10^{15}$ GeV $-$ 1 TeV.

The general expectation in spontaneously broken supergravity is that η should be of order unity. This is indeed what has been found so far in attempts to embed inflation within string theory [5]. Thus, the above conditions on η amount to a non-trivial tuning that is required for phenomenological viable scenarios of locked inflation with or without a subsequent phase of saddle inflation.

Case ii) does allow for $\eta \sim \mathcal{O}(1)$ if M is of order TeV scale. This is encouraging for attempts to embed inflation in string theory. It is important to keep in mind, however, that this assumes that density perturbations with the correct amplitude can be generated via the DGZ mechanism, even for such a low reheating temperature. For larger values of M, the model is forced towards $\eta \ll 1$, corresponding to the slow-roll inflationary regime.

We conclude with a comment on observational consequences of locked inflation. Here i) is the interesting case, since for case ii) the secondary "Long Saddle Inflation" phase erases all observational characteristics of the locked period. Locked inflation without subsequent saddle inflation either solves the standard cosmological problems in a single step with a low reheating scale or in a series of steps with a higher inflation scale, when N_{locked} (Eq. (2)) for each phase of inflation is too small on its own. In either case one still has to satisfy the constraint on η in Eq. (30) *at each step.* In multistage locked inflation this constraint does become weaker as M increases, as can be seen from Fig. 2. But the cost of this is that the constraint must be satisfied at the end of each of the multiple phases of locked inflation. Instead of needing to solve a stringent constraint once as is the case with locked inflation at low mass scale, one must solve a weaker constraint several times in order to construct a viable model.

If there are several phases of locked inflation, this could have immediate observational consequences. In DK's proposal, perturbations produced during locked inflation arise via the previously mentioned DGZ mechanism, whereby the amplitude of the perturbations depends on the coupling of the inflaton field to another field. In general, this coupling and the resulting spectrum of perturbations will have a different amplitude during each phase of locked inflation. Consequently, the resulting perturbation spectrum could contain discontinuities corresponding to the different phases of locked inflation. While there is no guarantee that one of these discontinuities will appear in the portion of the spectrum probed by observations, if M is large enough the amount of inflation produced during each phase is small enough to make this unavoidable. This argument is similar to that considered in [19] for a single field model where the potential contains a number of steps – putting one "feature" in the perturbation spectrum requires tuning, whereas adding many features makes it likely that at least one will fall in the range of k accessible to cosmological measurements. This topic deserves further study, particularly if the evidence for a running spectral index seen in the first year data of WMAP survives.

Acknowledgements

We thank C. Burgess, G. Dvali, A. Hamilton, S. Kachru, A. Linde and E. Weinberg for useful discussions. This work is supported in part by the DOE grants DE-FG02-92ER-40704 (RE) and DE-FG02-92ER-40699 (KS), the CU Academic Quality Fund and the Ohrstrom Foundation (JK).

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